

**Entrance to Grade 12 University Advanced Functions (MHF4U)  
or Mathematics of Data Management (MDM4U)**

*Scrap paper is available but write your final solution clearly in the space provided*

Let the integers be represented by  $x$  and  $(-31-x)$ .

$$x(-31-x) = 240$$

$$-31x - x^2 = 240$$

$$x^2 + 31x + 240 = 0$$

$$(x+15)(x+16) = 0$$

$$x = -15, -16$$

If  $x = -15$ , the second integer is  $(-31-x) = -31+15 = -16$ .

If  $x = -16$ , the second integer is  $(-31-x) = -31+16 = -15$ .

[1] The two consecutive integers are  $-15$  and  $-16$ .

a. When  $t = 0$ ,  $h = 5$ . The ball is released at 5 cm.

b.  $h = 5 + 50t - 5t^2$

$$= -5(t^2 - 10t + 25) + 5 + 125$$

$$= -5(t-5)^2 + 130$$

The maximum height is 130 m.

c. The ball hits the ground when  $h = 0$ .

$$5 + 50t - 5t^2 = 0$$

$$5t^2 - 50t - 5 = 0$$

$$t^2 - 10t - 1 = 0$$

$$t = \frac{10 \pm \sqrt{100+4}}{2}$$

$$= \frac{10 \pm \sqrt{104}}{2}$$

$$\doteq 10.099 \text{ or } -0.099$$

[2] Time cannot be a negative. The ball stays in the air for 10.1 s and Margarita's team can win a prize.

$$\begin{aligned} \text{a. } 3^{x^2+2x} &= 27 \\ 3^{x^2+2x} &= 3^3 \\ x^2 + 2x &= 3 \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \\ x &= -3, +1 \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{1}{4}(2)^{x^2} &= 2^x \\ 2^{-2}(2)^{x^2} &= 2^x \\ 2^{x^2-2} &= 2^x \\ x^2 - 2 &= x \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, -1 \end{aligned}$$

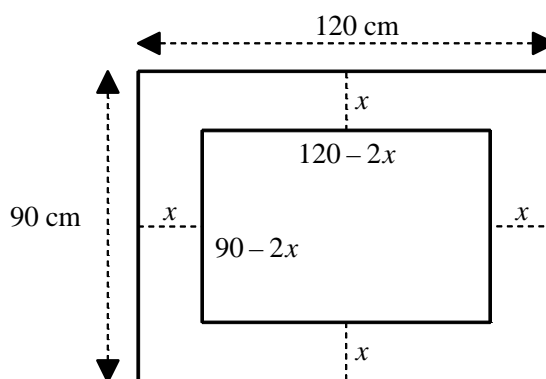
$$\begin{aligned} \text{b. } 2^{x^2} \times 2^{3x} &= \frac{1}{4} \\ 2^{x^2+3x} &= 2^{-2} \\ x^2 + 3x &= -2 \\ x^2 + 3x + 2 &= 0 \\ (x+2)(x+1) &= 0 \\ x &= -2, -1 \end{aligned}$$

$$\begin{aligned} \text{d. } 3^x \times 81 &= 3^{5-x^2} \\ 3^x \times 3^4 &= 3^{5-x^2} \\ 3^{x+4} &= 3^{5-x^2} \\ x+4 &= 5-x^2 \\ x^2 + x - 1 &= 0 \\ x &= \frac{-1 \pm \sqrt{1+4}}{2} \\ &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

[3]

Let  $x$  represent the width of the frame.

$$\begin{aligned} (90-2x)(120-2x) &= 8800 \\ 10\,800 - 180x - 240x + 4x^2 &= 8800 \\ 4x^2 - 420x + 2000 &= 0 \\ x^2 - 105x + 500 &= 0 \\ (x-100)(x-5) &= 0 \\ x &= 100, 5 \end{aligned}$$



If  $x = 100$ , the length and width of the mirror would be negative.

[4] Therefore, the width of the frame is 5 cm.

$$\begin{aligned} \text{a. } 3^x + 6 &= 87 \\ 3^x &= 81 \\ 3^x &= 3^4 \\ x &= 4 \end{aligned}$$

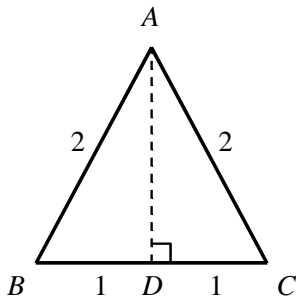
$$\begin{aligned} \text{c. } 4^{x+1} - 22 &= 42 \\ 4^{x+1} &= 64 \\ 4^{x+1} &= 4^3 \\ x+1 &= 3 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} \text{b. } 5(2^x) - 160 &= 0 \\ 2^x - 32 &= 0 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} \text{d. } \frac{3^{x-1}}{6} &= \frac{81}{2} \\ 3^{x-1} &= 243 \\ 3^{x-1} &= 3^5 \\ x-1 &= 5 \\ x &= 6 \end{aligned}$$

[5]

a.



b.  $DC = 1$

c.  $2^2 = AD^2 + 1^2$

$AD^2 = 3$

$AD = \sqrt{3}$

d.  $\angle C = 60^\circ$

e.  $\angle DAC = 30^\circ$

f.  $\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

g.  $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$

[6]

a.  $x^2 + 81 = 0$

$x^2 = -81$

$x = \pm\sqrt{-81}$

$= \pm 9i$

c.  $x^2 + 15 = -12$

$x^2 = -27$

$x = \pm\sqrt{-27}$

$= \pm 3\sqrt{3}i$

b.  $3(x-3)^2 = x^2 - 3$

$3(x^2 - 6x + 9) = x^2 - 3$

$3x^2 - 18x + 27 = x^2 - 3$

$2x^2 - 18x + 30 = 0$

$x^2 - 9x + 15 = 0$

$x = \frac{9 \pm \sqrt{21}}{2}$

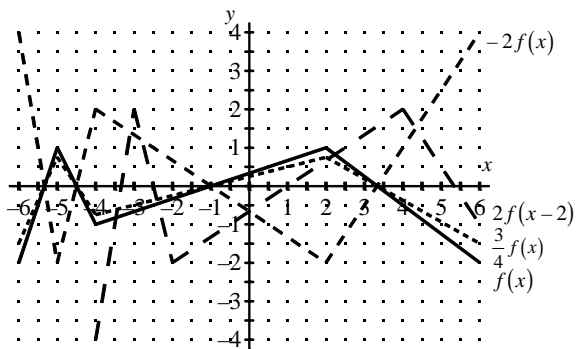
d.  $(x+2)(x+5) = (x-4)^2$

$x^2 + 7x + 10 = x^2 - 8x + 16$

$15x = 6$

$x = \frac{2}{5}$

[7]



[8]